Parametric Forecasts of Australian Yield Curves

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Abstract This paper is concerned with forecasting the profile of interest rates over time. Conventional time series methods have been generally unsuccessful in forecasting interest rates, with fitted ARIMA models being close to random walks. The method proposed here is to forecast the whole of the yield curve, from which forecasts of individual rates may be extracted. The methodology is motivated by the following observations. Over time, the efficient markets hypothesis suggests that all available information about a bond will be rapidly incorporated into its market price (assuming free information, zero transactions cost and rational behaviour). One implication of this hypothesis is that spot rates will follow a random walk. This suggests that the shape of the yield curve may fluctuate randomly. Thus, a yield curve estimated as a specific functional form has parameters that may follow a random walk. If yield curves or their estimated parameters do not follow random walks, then arbitrage possibilities may exist. In the maturity dimension, bonds of close maturity are close substitutes, so market efficiency arguments require that the yield curve be "smooth" across maturities. If an observed yield for a specific maturity lies below the yield curve, then this could be sold and be replaced by lower priced (higher yield) bonds, thereby improving the average yield of a portfolio while maintaining the same duration. Hence, interest rates may be random walks over time, but yields will change smoothly with maturity at a particular point in time. This paper exploits this regularity to obtain improved forecasts of interest rates.

1.1 Introduction

The method proposed to exploit the smoothness across maturities of the yield curve can be summarised by the following steps:

1. Estimate a parametric form for the yield curve at time t, with estimated parameters x_t.
2. Fit a time series model to the sequence $\{x_1, x_2, \ldots, x_T\}$, for example a VAR(1) $x_t = Ax_{t-1} + u_t$.
3. Use the model to forecast x_{T+k}
4. Construct the forecasted yield curve at time T+k.
5. Extract the implied forecast of the spot interest rate.

In the next section, methods for fitting yield curves are outlined, and then the proposed method is applied to forecasting the yields on Australian Government Bonds. The forecasting performance is then evaluated.

1.2 Fitting Yield Curves

Attempts to fit yield curves have a long history. The methods used include freehand curves (Durand[1941]), polynomial and exponential splines (McCulloch[1971], Shea[1984, 1985] inter alia) and parametric forms based on polynomial regressions (Echols and Elliot [1976], Chambers et al [1984] inter alia). Lau [1983] and Nelson and Siegel [1987] have estimated parsimonious models of the yield curve based on Laguerre functions. The method chosen to fit the yield curve will to some extent be governed by the ultimate use of the estimated curve. If it required as a simple descriptive device (such as required in the financial press) then most methods will provide a good fit to the observed yields. Nelson and Siegel's purpose was to extract estimates of the long term bond rate by extrapolating beyond their observed maturities, so that splines and polynomials that tend to bend towards the end of the maturity ranges were unsuitable functional forms, and this led to their choice of the Laguerre functional form that asymptotes at long maturities.

The functional forms for the yield curve considered in this study were the Laguerre form used by Lau [1983] and Nelson and Siegel [1987] and a simple polynomial functional form:

$$y(m) = \beta_1 \, exp\left(-\frac{m}{\beta_2}\right) + \beta_3\left(\frac{m}{\beta_2}\right) exp\left(-\frac{m}{\beta_2}\right) + \beta_4 \quad (1)$$

$$y(m) = \beta_1 m + \beta_2 m^{-1} + \beta_3 \, log_e(m) + \beta_4 \quad (2)$$

where y(m) is the yield, m is the maturity of a bond, and the β's are parameters to be estimated.

1.3 Australian Government Bond Yield Curves

The data used for this study were extracted from the Bond Sheets of a leading Australian bond dealer. These sheets provided the yield to maturity for all Commonwealth Government Bonds from week 12, 1987 to week 39, 1988, giving in all 80 weekly observations. Maturities range up to 18 years. A number of "outliers" were identified and removed from the analysis. These were the low coupon bonds (≤ 7%) issued before November 1977, and the 12.5% March 1997 bond issued in October 1986, whose yields to maturity were "too low". If after tax yields, rather than gross yields were used in the analysis then these observations may no longer be outliers. The models were estimated using the non-linear least squares option in SHAZAM (see White [1978]). A major problem was encountered in that the estimated Laguerre models were occasionally overparameterised, with large changes in β_3 giving essentially the same fit. In these cases the parameter was constrained to zero (a restriction supported by likelihood ratio tests). The Laguerre functional forms actually fit the observed yields better, and are in fact preferred functional forms to the polynomial form when non-nested hypothesis tests are performed. The over-parameterisation problem makes the Laguerre form unsuitable for the forecasting exercise, so the remainder of this paper focuses on the results from estimating the polynomial form. The polynomial form provides a reasonable within sample fits, with the average standard error of fit over the 80 periods being 14.52 basis points (the smallest is 8.30 basis points and the largest is 30.32 basis points). The fit of the estimated model is illustrated in Figures 1-4 using the last four weeks of the sample data. The estimated parameters and the standard error of estimate for each fitted yield curve are presented in the Appendix in Table 3. Although all the parameters are clearly not needed at each point in time, the estimates of β_1, β_2, β_3 and β_4 are significant at the 1% level for 27, 29, 69 and 80 time periods respectively.

The parameter estimates show marked time dependence, as is evident by the autocorrelation functions reported in Table 4. A vector autoregressive model of order 3 (VAR(3)) was fitted to the sequence of four estimated parameters of β_1, β_2, β_3 and β_4 as well as the standard error of estimate s. There are some significant interactions between the variables that are summarised in Table 1:

Table 1: Interactions in VAR(3)

Equation for	Determined by	R^2
β_1	β_1, β_3 and β_4	0.80
β_2	β_3	0.64
β_3	β_2, β_3 and β_4	0.94
β_4	β_2, β_3 and β_4	0.93
s	s	0.50

A restricted VAR model (RVAR) was also fitted to the estimated parameters, with the optimal model being chosen using the Schwartz criterion. This resulted in a VAR of order one with only seven parameters being chosen. As an illustration, the parameters of the preferred model, estimated over all eighty observations are given in Table 2.

Table 2: Estimated RVAR parameters

	β_1	β_2	β_3	β_4	s
β_1	0.770	0	0	0	0
β_2	0	0	0.231	0	0
β_3	-0.122	0	0.982	0	0
β_4	0	0	0	0.955	0
s	0	0	-0.203	0	0.449

The forecasting performance of these models is evaluated using two metrics - the ability to forecast the actual yields on bonds and the ability to forecast the parameters of estimated yield curves. The last eight observations are used to evaluate the forecasting performance. The estimated VAR models were used to obtain forecasts of the parameters for one and two step horizons, and all models were re-estimated prior to forecasting. The estimated parameters were used to derive a forecasted yield curve.

Figures 1 to 4 illustrates, for the last four weeks of the sample period, the actual yields, the polynomial model fitted to the actual yields, and the one and two step forecasted yields derived from the VAR(3) model. The forecasted yield curves are in the "ball park" and appear to converge to the fitted curve as the forecast horizon shortens.

Some summary statistic for the performance of the VAR models and the naive model for forecasting the parameter values of the fitted yield curves from week 32/88 to week 39/88 are presented in Table 6. One and two step forecast horizons are presented and all models were re-estimated prior to forecasting. The fitted parameter values are treated as the true values in this evaluation, and there appears to be a small gain in terms of root mean square error in using the VAR models over the naive model.

Some summary statistics for the forecasting performance of the two VAR models and a naive no-change model for the actual yields to maturity are reported in Table 5. In terms of the root mean square error of the forecasts, there appears to be no overall gain in the forecasting performance in using the VAR models over the naive forecasting model. For one step forecasts the VAR3 (RVAR) forecasts have a smaller root mean square error than the naive forecasts for 2 (2) weeks, whereas for the two step forecasts the VAR3 (SVAR) root mean square error is smaller for 5 (1) weeks.

1.4 Conclusions

A methodology has been developed to obtain forecasts of the whole yield curve. Using this method, the forecasts appear to perform at least as well as the naive no-change forecasts, and the method warrants further evaluation. Areas worth further investigation include considering richer functional forms used to model the yield curve need to be considered and the time series methodology used to forecast the parameter values. In the end, a proper evaluation of the method requires a careful specification of the end use of the forecasts.

1.5 References

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1.6 Appendix

Table 3: Estimated Parameters from Polynomial Functional Form

Week	β_1	β_2	β_3	β_4	s	R^2	N
12/87	0.094*	-0.059*	-0.778*	14.623*	0.125	0.994	89
13/87	0.061*	-0.028*	-0.565*	14.370*	0.100	0.938	89
14/87	0.041*	-0.017*	-0.482*	14.203*	0.102	0.933	89
15/87	0.056*	-0.088*	-0.565*	14.301*	0.109	0.897	88
16/87	0.055*	-0.082*	-0.521*	13.953*	0.105	0.879	88
17/87	0.033*	-0.032*	-0.336*	13.942*	0.085	0.860	88
18/87	0.001	-0.007#	-0.158*	13.554*	0.083	0.786	88
19/87	-0.037*	-0.165*	0.112	13.137*	0.116	0.634	87
20/87	-0.004	-0.040	0.006	13.074*	0.111	0.079	87
21/87	0.011	-0.075#	-0.083	13.431*	0.115	0.097	87
22/87	-0.008	-0.003	0.077	13.354*	0.106	0.236	87
23/87	-0.019	-0.019	0.202*	13.136*	0.150	0.578	87
24/87	-0.027#	-0.025	0.276*	12.679*	0.136	0.769	87
25/87	-0.016	-0.023#	0.179*	13.140*	0.114	0.732	87
26/87	-0.037*	-0.008	0.245*	12.923*	0.159	0.570	87
27/87	-0.036#	0.001	0.262*	12.917*	0.212	0.462	87
28/87	-0.007	-0.076	0.164	12.755*	0.251	0.410	86
29/87	-0.018	-0.060	0.240*	12.671*	0.143	0.750	86
30/87	-0.028#	-0.022	0.350*	12.738*	0.137	0.835	86
31/87	-0.042#	0.055	0.547*	12.485*	0.187	0.830	86
32/87	-0.063*	0.060#	0.662*	12.260*	0.166	0.889	86
33/87	-0.004	0.024	0.376*	12.287*	0.128	0.905	87
34/87	0.012	0.035#	0.371*	12.143*	0.187	0.845	87
35/87	0.020	0.021*	0.332*	12.272*	0.148	0.898	87
36/87	0.034#	0.058	0.243*	12.216*	0.163	0.826	86
37/87	0.026	0.086*	0.345*	11.656*	0.205	0.802	86
38/87	0.035#	0.059*	0.315*	11.428*	0.180	0.855	86
39/87	0.071*	0.025#	0.166*	11.615*	0.165	0.878	86
40/87	0.070*	0.017#	0.145#	11.680*	0.235	0.760	86
41/87	0.031	-0.030	0.322*	11.938*	0.174	0.894	85
42/87	0.010	0.116#	0.463*	12.451*	0.197	0.850	85
43/87	-0.004	-0.091*	0.381*	13.453*	0.149	0.927	85
44/87	-0.047*	-0.063#	0.691*	12.798*	0.146	0.962	85
45/87	-0.099*	0.229*	1.415*	11.390*	0.303	0.927	85
46/87	-0.105*	0.110*	1.203*	11.746*	0.194	0.960	85
47/87	-0.056#	-0.085	1.126*	11.437*	0.170	0.967	83
48/87	-0.126*	0.354*	1.671*	10.820*	0.180	0.976	83

= Significant at 5% level, * = significant at 1% level

Table 3 continued: Estimated Parameters from Polynomial Functional Form

Week	β_1	β_2	β_3	β_4	s	R^2	N
49/87	-0.113*	0.336*	1.643*	10.678*	0.207	0.971	83
50/87	-0.077*	0.241*	1.454*	10.546*	0.175	0.979	83
51/87	0.006	0.094#	0.902*	10.868*	0.145	0.979	83
52/87	0.018	0.076#	0.814*	10.984*	0.145	0.978	83
1/88	0.004	0.089*	0.878*	10.947*	0.148	0.978	83
2/88	0.016	0.083*	0.827*	10.891*	0.164	0.973	83
3/88	-0.001	0.112*	0.902*	10.557*	0.152	0.976	83
4/88	0.017	0.051	0.697*	10.757*	0.149	0.970	82
5/88	-0.012	0.049	0.809*	10.962*	0.133	0.978	82
6/88	-0.033#	0.056#	0.849*	11.042*	0.159	0.968	82
7/88	-0.047*	0.065*	0.861*	11.000*	0.139	0.973	82
8/88	-0.016	0.041*	0.642*	11.157*	0.163	0.950	82
9/88	-0.015	0.038*	0.670*	10.917*	0.148	0.964	82
10/88	0.008	0.015*	0.579*	10.911*	0.174	0.952	82
11/88	0.008	0.021	0.499*	10.871*	0.144	0.952	81
12/88	0.052*	0.017	0.444*	10.636*	0.168	0.953	81
13/88	0.054*	0.015*	0.406*	10.617*	0.171	0.946	81
14/88	0.071*	-0.005	0.335*	10.457*	0.149	0.951	78
15/88	0.078*	-0.008	0.196*	10.857*	0.140	0.934	78
16/88	0.049*	0.024	0.262*	10.744*	0.115	0.939	77
17/88	0.020	0.070	0.533*	10.496*	0.122	0.963	77
18/88	-0.014	-0.067	0.542*	11.331*	0.134	0.961	77
19/88	-0.031	0.015	0.436*	11.475*	0.127	0.906	78
20/88	-0.001	0.038	0.176*	11.844*	0.104	0.718	78
21/88	-0.027	0.044	0.270*	12.207*	0.108	0.744	78
22/88	-0.037*	0.044	0.357*	12.008*	0.120	0.825	78
23/88	-0.091*	0.063*	0.574*	11.771*	0.176	0.795	78
24/88	-0.032	0.042*	0.326*	11.732*	0.147	0.721	78
25/88	0.000	0.015	0.257*	11.641*	0.136	0.838	78
26/88	0.002	0.007	0.110*	11.758*	0.103	0.624	78
27/88	-0.003	0.004	0.088*	11.796*	0.105	0.471	78
28/88	-0.011	0.004*	0.111*	11.854*	0.101	0.504	78
29/88	0.002	-0.018	-0.022	12.145*	0.105	0.038	75
30/88	0.008	-0.008	-0.040	12.138*	0.112	0.016	75
31/88	-0.001	-0.003	-0.001	12.130*	0.108	0.009	75
32/88	-0.012	0.003	0.036	12.378*	0.107	0.042	75
33/88	-0.036*	0.022	0.073	12.333*	0.102	0.361	74
34/88	-0.032*	0.000	-0.066	12.582*	0.120	0.722	74
35/88	-0.016	-0.012	-0.178*	12.606*	0.124	0.806	74
36/88	-0.025	-0.004	-0.166*	12.614*	0.139	0.811	74
37/88	0.016	-0.077*	-0.416*	12.883*	0.136	0.859	74
38/88	-0.004	-0.018	-0.347*	12.882*	0.125	0.915	73
39/88	0.006	-0.062*	-0.522*	13.396*	0.132	0.937	73

= Significant at 5% level, * = significant at 1% level

Table 4: Autocorrelation Coefficients

Lag	β_1	β_2	β_3	β_4	s
1	0.770	0.476	0.884	0.904	0.568
2	0.553	0.283	0.776	0.816	0.416
3	0.330	0.444	0.695	0.747	0.474
4	0.166	0.374	0.617	0.669	0.469
5	-0.044	0.181	0.519	0.606	0.425
6	-0.271	0.190	0.424	0.546	0.284
7	-0.390	0.166	0.365	0.515	0.219
8	-0.394	0.142	0.330	0.497	0.278
9	-0.353	0.140	0.290	0.469	0.240
10	-0.261	0.136	0.252	0.437	0.129

Table 5: Summary Statistics for Parameter forecasts for the weeks 32/88 to 39/88

Method	Statistic	β_1	β_2	β_3	β_4	s
1-Step						
Naive	Mean	-0.001	0.007	0.065	-0.158	-0.003
	SD	0.020	0.038	0.111	0.181	0.009
	RMSE	0.020	0.038	0.129	0.241	0.010
VAR(3)	Mean	0.008	-0.021	-0.038	0.024	-0.009
	SD	0.018	0.039	0.118	0.220	0.010
	RMSE	0.020	0.038	0.129	0.241	0.010
RSVAR(1)	Mean	0.001	-0.018	0.062	-0.123	-0.008
	SD	0.018	0.024	0.098	0.163	0.008
	RMSE	0.018	0.030	0.116	0.204	0.011
2-Step						
Naive	Mean	0.001	0.009	0.103	-0.251	-0.004
	SD	0.022	0.027	0.117	0.124	0.013
	RMSE	0.022	0.029	0.156	0.280	0.013
VAR(3)	Mean	0.017	-0.033	-0.080	0.057	-0.014
	SD	0.016	0.030	0.108	0.169	0.012
	RMSE	0.023	0.045	0.134	0.178	0.018
RVAR(1)	Mean	0.003	-0.015	0.117	-0.233	-0.008
	SD	0.018	0.022	0.101	0.139	0.008
	RMSE	0.019	0.027	0.155	0.271	0.011

Note : The estimated parameters are treated as the true values in computing the root mean square error.

Table 6: Summary Statistics for forecasts of the Yield Curves Forecasts for the weeks 32/88 to 39/88

Method	Statistic	32	33	34	35	36	37	38	39
Fitted	Mean	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	SD	0.104	0.100	0.116	0.121	0.135	0.132	0.121	0.128
	RMSE	0.104	0.100	0.116	0.121	0.135	0.132	0.121	0.128
1-Step									
Naive	Mean	-0.246	0.114	-0.069	0.040	0.018	-0.110	-0.035	-0.295
	SD	0.113	0.119	0.168	0.137	0.143	0.154	0.181	0.172
	RMSE	0.271	0.165	0.181	0.143	0.145	0.189	0.184	0.341
VAR(3)	Mean	-0.259	0.186	-0.161	0.123	0.126	-0.012	0.085	-0.150
	SD	0.389	0.109	0.174	0.146	0.167	0.143	0.166	.158
	RMSE	0.467	0.216	0.209	0.191	0.209	0.144	0.187	.218
RVAR	Mean	-0.260	0.114	-0.048	0.059	0.026	-0.079	-0.009	-0.276
	SD	0.322	0.119	0.215	0.192	0.278	0.172	0.187	0.224
	RMSE	0.414	0.165	0.221	0.201	0.279	0.189	0.187	0.356
2-Step									
Naive	Mean	-0.244	-0.127	0.044	-0.028	0.061	-0.087	-0.140	-0.336
	SD	0.135	0.122	0.217	0.217	0.163	0.161	0.177	0.239
	RMSE	0.279	0.176	0.222	0.219	0.174	0.183	0.225	0.413
VAR(3)	Mean	-0.254	-0.077	0.056	-0.004	0.241	0.118	0.088	-0.098
	SD	0.617	0.119	0.173	0.164	0.202	0.146	0.144	0.157
	RMSE	0.667	0.142	0.182	0.164	0.315	0.188	0.169	0.185
RVAR	Mean	-0.246	-0.105	0.057	0.001	0.083	-0.040	-0.092	-0.299
	SD	0.332	0.158	0.257	0.300	0.301	0.199	0.255	0.268
	RMSE	0.413	0.190	0.263	0.300	0.312	0.203	0.271	0.402

Note: The actual observed yields are treated as the true values in computing the root mean square error.

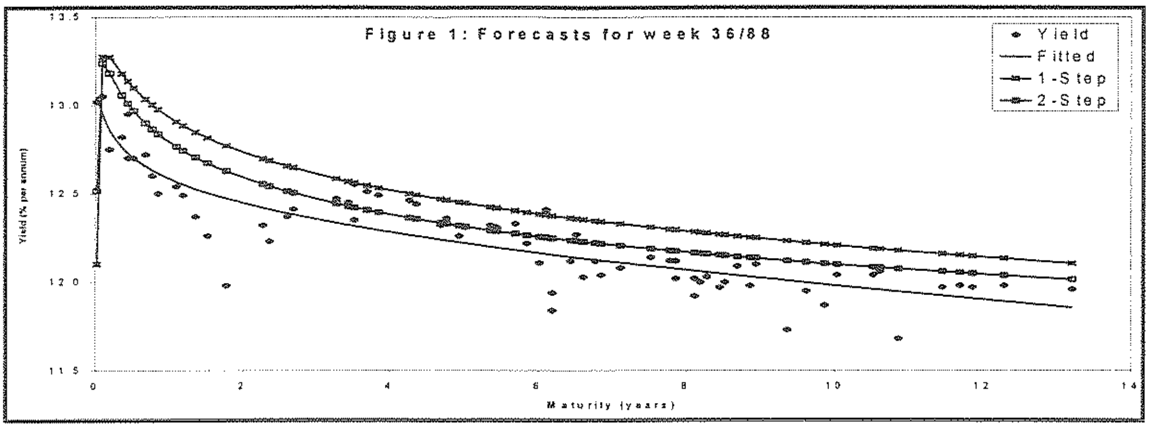
Figure 1: Forecasts for week 36/88
Yield
Fitted
1-Step
2-Step
Yield (% per annum)
13.5
13.0
12.5
12.0
11.5
0
2
4
6
8
10
12
14
Maturity (years)

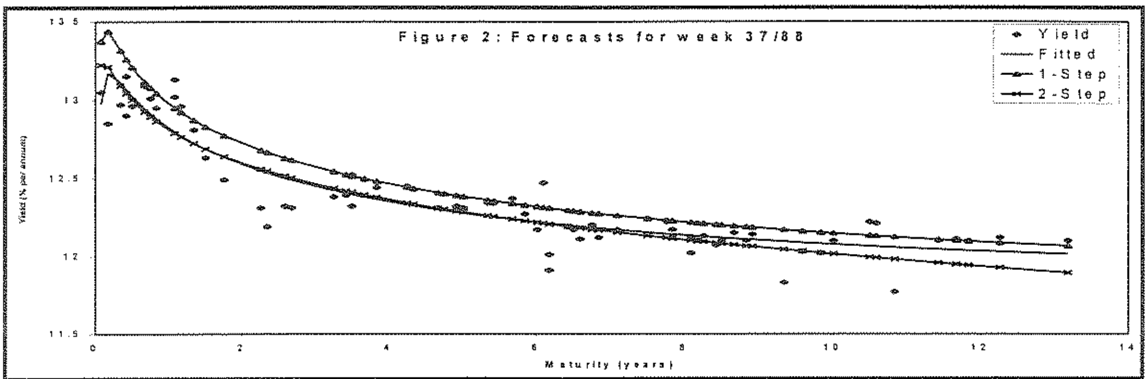
Figure 2: Forecasts for week 37/88
Yield
Fitted
1-Step
2-Step
Yield (% per annum)
13.5
13
12.5
12
11.5
0
2
4
6
8
10
12
14
Maturity (years)

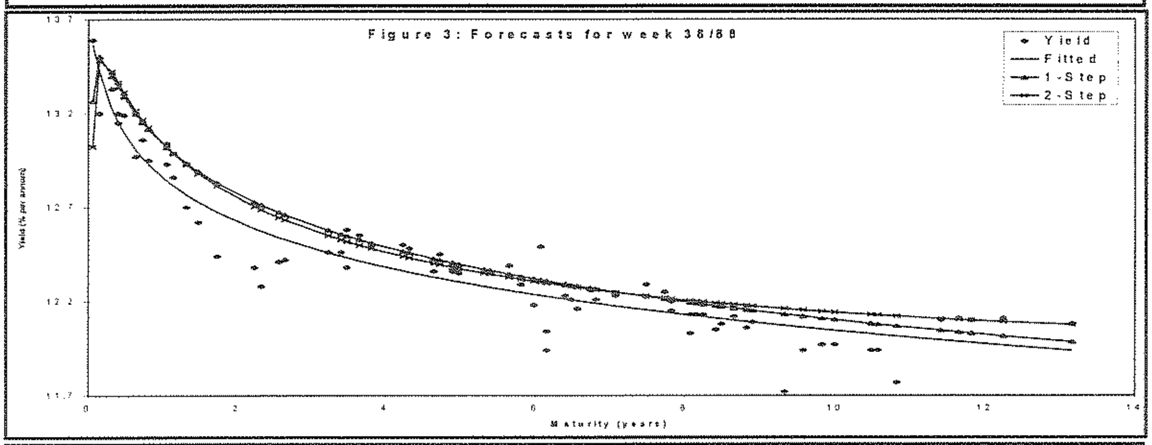
Figure 3: Forecasts for week 38/88
Yield
Fitted
1-Step
2-Step
Yield (% per annum)
13.7
13.2
12.7
12.2
11.7
0
2
4
6
8
10
12
14
Maturity (years)

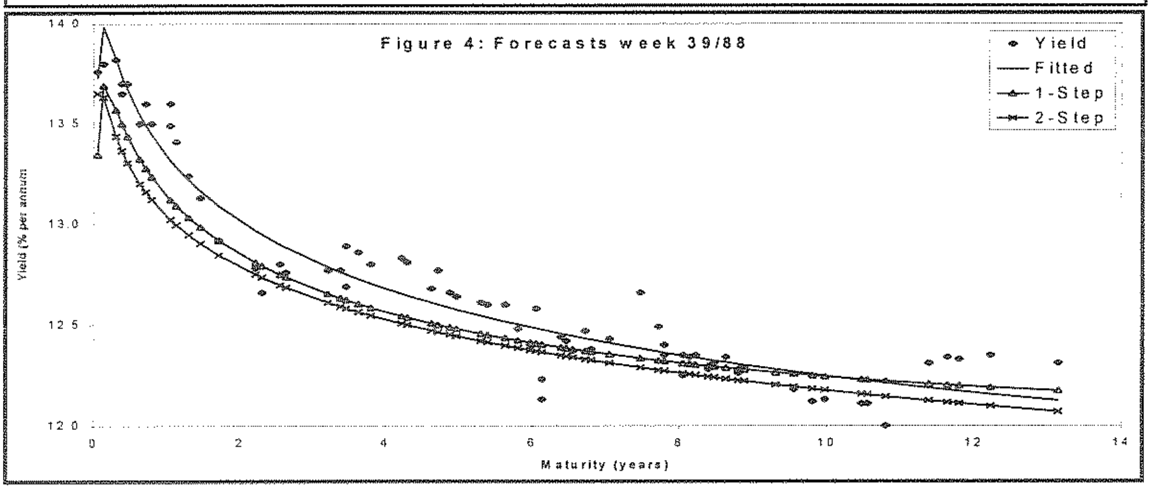
Figure 4: Forecasts week 39/88
Yield
Fitted
1-Step
2-Step
Yield (% per annum)
14.0
13.5
13.0
12.5
12.0
0
2
4
6
8
10
12
14
Maturity (years)